

Ramond-Ramond Central Charges in the Supersymmetry Algebra of the Superstring

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The free action for the massless sector of the Type II superstring was recently constructed using closed RNS superstring field theory. The supersymmetry transformations of this action are shown to satisfy an N=2 D=10 SUSY algebra with Ramond-Ramond central charges.

1. Introduction

One of the most important reasons for believing superstring theory is related to a higher-dimensional theory is the fact that non-perturbative superstring states called D -branes carry Ramond-Ramond charge[1]. This provides a connection between eleven-dimensional Kaluza-Klein states and superstring theory since Ramond-Ramond charge is related to momentum in the eleventh direction. In this letter, it is shown that R-R central charges are present in the supersymmetry algebra of the superstring, providing an even more direct connection with a higher-dimensional theory.

Ramond-Ramond central charges are usually assumed to be absent in the spacetime-supersymmetry algebra of the superstring. This assumption is based on three arguments: 1) No perturbative superstring states carry Ramond-Ramond charge; 2) The left and right-moving spacetime-supersymmetry generators anti-commute with each other in the RNS formalism;¹ and 3) There is no Wess-Zumino term for Ramond-Ramond fields in the standard GS sigma model.[3]

Although all of these arguments are correct, it will be shown here that R-R central charges are nevertheless present in the SUSY algebra of the superstring. Because of the first argument, these charges can be detected only through the abelian transformation of the R-R gauge field. Note that when “picture” is treated properly [4], zero-momentum R-R gauge fields are present in the string field, although they decouple from all perturbative superstring states. Using superstring field theory, the transformation of the R-R gauge field is enough to construct the global R-R charge out of RNS matter and ghost worldsheet variables. When acting on perturbative superstring states, this charge is BRST-trivial, which is not ruled out by the second argument since the supersymmetry algebra in the RNS description closes only up to gauge transformations and equations of motion. Finally, the dependence of the R-R charge on worldsheet ghosts explains why the third argument is inapplicable. In the standard GS sigma model, worldsheet ghosts are not understood, as can be seen by the absence of a Fradkin-Tseytlin term[5] which should couple the spacetime dilaton to worldsheet curvature.

Because of picture-changing difficulties, a free action for the R-R fields of the Type II superstring has only recently been constructed[4]. In the massless sector, the R-R string field depends on an “electric” gauge field $A_{(+)}^{\alpha\beta}$, a “magnetic” gauge field $A_{(-)}^{\alpha\beta}$, and an

¹ The left-moving with left-moving SUSY generators anti-commute to give the usual momentum and NS-NS one-brane charge, but no NS-NS five-brane charge.[2]

infinite set of auxiliary fields $F_{(n)}^{\alpha\beta}$ for $n=1$ to ∞ . (α and β are 16-component $\text{SO}(9,1)$ spinor indices whose chirality depends if one is discussing Type IIA or Type IIB.) This set of massless fields can be understood using the usual RNS formalism if the R-R vacuum state is chosen to be annihilated by γ_L^0 and β_R^0 (as opposed to β_L^0 and β_R^0) where $[\beta_L^0, \gamma_L^0]$ and $[\beta_R^0, \gamma_R^0]$ are the zero modes of the left and right-moving bosonic ghosts.

After adding the NS-R, R-NS, and NS-NS massless sectors, the action is invariant under coordinate and $b_{\mu\nu}$ gauge transformations, under local $N=2$ $D=10$ supersymmetry transformations, and under R-R gauge transformations of $A_{(+)}^{\alpha\beta}$ and $A_{(-)}^{\alpha\beta}$. It will be shown here that the commutator of two supersymmetry transformations gives a coordinate transformation, a $b_{\mu\nu}$ gauge transformation, and a R-R gauge transformation. This means that the SUSY algebra of the Type II superstring contains R-R central charges as well as the usual NS-NS one-brane central charge.

Note that even without knowing superstring field theory, the presence of Ramond-Ramond central charges could have been predicted based on the role of the dilaton as a conformal compensator[6]. In $N=2$ $D=4$ supergravity coupled to vector multiplets, it is well-known that the $N=2$ $D=4$ SUSY algebra contains central charges of the form[7]

$$[\delta_{q_i^\alpha}(u_i^\alpha), \delta_{q_j^\beta}(v_j^\beta)] = \sum_n \delta_{C_{(n)}}(W_{(n)} u^{i\alpha} v_\alpha^j \epsilon_{ij}) \quad (1.1)$$

where i is an $\text{SU}(2)$ index, $W_{(n)}$ is the expectation value of the complex scalar in the n^{th} vector multiplet, $\delta_{q_i^\alpha}(u_i^\alpha)$ is the SUSY transformation with parameter u_i^α , and $\delta_{C_{(n)}}(\rho_{(n)})$ is the gauge transformation of the n^{th} gauge field with parameter $\rho_{(n)}$.

One of these vector multiplets is the vector compensator multiplet and, as shown in [6], the expectation value of the scalar in this multiplet is $\langle e^{-\phi} \rangle = g^{-1}$ where ϕ is the dilaton and g is the string coupling constant. So one expects R-R central charges in the Type II superstring, at least after compactifying to four dimensions.² This suggests that a proper understanding of conformal compensators in ten dimensions will help in understanding the role of R-R central charges.

² The g^{-1} factor in this central charge can be explained by recalling that the gauge field in the conformal compensator, \hat{A}^μ , is related to the R-R gauge field in superstring field theory, A^μ , by $\hat{A}^\mu = e^{-\phi} A^\mu$. (This can be seen from the fact that the kinetic term for \hat{A}_μ carries no $e^{-2\phi}$ factor in the tree-level effective action.) For this reason, there is no g^{-1} factor in the SUSY algebra when R-R charge is defined with respect to A^μ rather than \hat{A}^μ .

2. Superstring Field Theory

2.1. The massless R-R sector

The massless R-R contribution to the free Type II superstring action was recently constructed using closed superstring field theory.[4] Although “non-minimal” fields[8] were needed for this construction, the equations of motion and gauge invariances can be easily analyzed using the standard RNS worldsheet variables.

Physical closed superstring states are described by fields $|\Phi\rangle$ of zero total ghost-number where total ghost number is $g_L + g_R - 2$, $g_{L/R} = \oint d\sigma(b_{L/R}c_{L/R} - \eta_{L/R}\xi_{L/R})$, and the bosonic ghosts have been fermionized as $\beta = \partial\xi e^{-\phi}$ and $\gamma = \eta e^{\phi}$. (The -2 is present so that physical states carry zero ghost number, implying that the $SL(2)$ -invariant vacuum carries -2 total ghost-number.) Note that this definition of ghost-number is slightly modified from the standard one, $g = \oint d\sigma(bc - \partial\phi)$, although they agree at zero picture. (Picture is defined by $P = \oint d\sigma(\eta\xi - \partial\phi)$.) The modified definition of g is necessary in order that the spacetime-supersymmetry generators carry zero ghost number.

These string fields must satisfy the constraints $b_-^0|\Phi\rangle = L_-^0|\Phi\rangle = \eta_{L/R}^0|\Phi\rangle = 0$ and are defined up to the gauge transformation $\delta|\Phi\rangle = Q|\Lambda\rangle$ where $Q = Q_L + Q_R$ is the BRST charge, $b_-^0 = b_L^0 - b_R^0$, $L_-^0 = L_L^0 - L_R^0$ is the difference of the left and right-moving energies, $\eta_{L/R}^0|\Phi\rangle = 0$ implies no dependence on $\xi_{L/R}^0$, and $|\Lambda\rangle$ is a gauge field of -1 total ghost-number satisfying $L_-^0|\Lambda\rangle = b_-^0|\Lambda\rangle = \eta_{L/R}^0|\Lambda\rangle = 0$. The equation of motion is $Q|\Phi\rangle = 0$.

Finally, there is a constraint coming from the restriction to a single picture. Surprisingly, different choices of left and right-moving picture, P_L and P_R , give different sets of on-shell fields.³ For example, it will be found that there are on-shell constant modes in the R-R sector if $(P_L, P_R) = (-3/2, -1/2)$, but not if $(P_L, P_R) = (-1/2, -1/2)$. Since one wants to reproduce the equations of motion from reference [4] where on-shell zero-momentum R-R fields were found using an action principle, (P_L, P_R) will be chosen to be $(-3/2, -1/2)$ in the R-R sector.

³ The usual argument for picture independence of BRST cohomology comes from the fact that $YZ = 1$ where $Y = c\partial\xi e^{-2\phi}$, $Z = \{Q, \xi\}$, and $\partial Y, \partial Z$ are BRST trivial. For the open superstring, $Q|V\rangle = 0$ and $|V\rangle \neq Q|\Lambda\rangle$ implies that $QZ|V\rangle = 0$ and $Z|V\rangle \neq Q|\hat{\Lambda}\rangle$ (since $Z|V\rangle = Q|\hat{\Lambda}\rangle$ implies $|V\rangle = QY|\hat{\Lambda}\rangle$). But for the closed superstring, $Q|V\rangle = 0$ and $|V\rangle \neq Q|\Lambda\rangle$ does not imply $QZ_L|V\rangle = 0$ and $Z_L|V\rangle \neq Q|\hat{\Lambda}\rangle$. This is because Y_L and b_- do not commute, so although $Z_L|V\rangle = Q|\hat{\Lambda}\rangle$ implies $|V\rangle = QY_L|\hat{\Lambda}\rangle$, $b_-^0 Y_L|\hat{\Lambda}\rangle$ is not necessarily zero even when $b_-^0|\hat{\Lambda}\rangle = 0$. Therefore, closed superstring BRST cohomology can depend on the choice of picture.

This means that R-R states can be constructed using non-negative modes acting on a vacuum $|0\rangle_{R-R}^{\alpha\beta}$ which satisfies

$$b_L^0|0\rangle_{R-R}^{\alpha\beta} = b_R^0|0\rangle_{R-R}^{\alpha\beta} = \gamma_L^0|0\rangle_{R-R}^{\alpha\beta} = \beta_R^0|0\rangle_{R-R}^{\alpha\beta} = 0. \quad (2.1)$$

(The picture $(P_L, P_R) = (-\frac{1}{2}, -\frac{1}{2})$ would correspond to a vacuum satisfying $\beta_L^0|0\rangle_{R-R}^{\alpha\beta} = \beta_R^0|0\rangle_{R-R}^{\alpha\beta} = 0$.) In terms of the SL(2)-invariant vacuum, $|0\rangle_{R-R}^{\alpha\beta}$ is $c_L e^{-\frac{3}{2}\phi_L} \Sigma_L^\alpha c_R e^{-\frac{1}{2}\phi_R} \Sigma_R^\beta$ where $\Sigma_{L/R}^\alpha$ is the left and right-moving spin field of weight 5/8. The ψ_L^μ and ψ_R^μ zero modes are treated like SO(9,1) gamma-matrices which transform the bispinor indices on $|0\rangle_{R-R}^{\alpha\beta}$. Furthermore, all string states must be GSO-projected in the usual way.

Constructing all possible states of zero ghost-number, the massless R-R states of the Type II superstring are given by

$$|\Phi\rangle_{R-R} = \sum_{n=0}^{\infty} (A_{(n)}^{\alpha\beta}(x)(\beta_L^0)^n(\gamma_R^0)^n + F_{(n)}^{\alpha\beta}(x)c_+^0(\beta_L^0)^{n+1}(\gamma_R^0)^n) |0\rangle_{R-R}^{\alpha\beta}. \quad (2.2)$$

where $c_+^0 = c_L^0 + c_R^0$. Note that the GSO projection implies that the bispinor indices on $A_{(n)}^{\alpha\beta}$ and $F_{(n)}^{\alpha\beta}$ have different SO(9,1) chirality if n is even or odd. (In the above expressions, contracted spinor indices always have opposite SO(9,1) chiralities).

Using the gauge string field,

$$|\Lambda\rangle_{R-R} = \sum_{n=2}^{\infty} \Lambda_{(n)}^{\alpha\beta}(x)c_+^0(\beta_L^0)^n(\gamma_R^0)^{n-2}|0\rangle_{R-R}^{\alpha\beta}, \quad (2.3)$$

it is easy to check that $A_{(n)}^{\alpha\beta}$ can be algebraically gauged away for $n > 1$. The equation of motion $Q|\Phi\rangle = 0$, together with the requirement that only a finite number of fields are non-zero⁴, implies that the remaining fields satisfy[4]

$$\hat{F}_{(0)}^{\alpha\beta} = \partial^{\alpha\gamma} A_{(0)}^{\gamma\beta} - \partial^{\beta\gamma} A_{(1)}^{\alpha\gamma}, \quad \partial^{\beta\gamma} A_{(0)}^{\alpha\gamma} - \partial^{\alpha\gamma} A_{(1)}^{\gamma\beta} = 0, \quad (2.4)$$

$$\partial^{\alpha\gamma} \hat{F}_{(0)}^{\gamma\beta} = \partial^{\beta\gamma} \hat{F}_{(0)}^{\alpha\gamma} = 0, \quad F_{(2n+1)}^{\alpha\beta} = F_{(2n+2)}^{\alpha\beta} = 0$$

where $\hat{F}_{(0)}^{\alpha\beta} = F_{(0)}^{\alpha\beta} + \partial^{\alpha\gamma} A_{(0)}^{\gamma\beta}$ and $\partial^{\alpha\beta} = \Gamma_\mu^{\alpha\beta} \partial^\mu$. The fields and equations of (2.4) are the same as those of reference [4] and correspond to the Bianchi identities and equations of motion for a zero-form, two-form and four-form field strength (for Type IIA) or a one-form, three-form and self-dual five-form field strength (for Type IIB).

⁴ This requirement can be interpreted as a normalization condition on the string field[4].

The R-R fields $A_{(\pm)}^{\alpha\beta} = A_{(0)}^{\alpha\beta} \pm A_{(1)}^{\alpha\beta}$ play the role of “electric” and “magnetic” gauge fields with the gauge transformations,

$$\delta A_{(+)}^{\alpha\beta} = \partial^{\alpha\gamma} \rho_{(+)}^{\gamma\beta} + \partial^{\beta\gamma} \rho_{(+)}^{\alpha\gamma} \quad \delta A_{(-)}^{\alpha\beta} = \partial^{\alpha\gamma} \rho_{(-)}^{\gamma\beta} - \partial^{\beta\gamma} \rho_{(-)}^{\alpha\gamma}, \quad (2.5)$$

described by the gauge string field

$$|\Lambda\rangle_{R-R} = (\rho_{(0)}^{\alpha\beta}(x)\beta_L^0 + \rho_{(1)}^{\alpha\beta}(x)(\beta_L^0)^2\gamma_R^0 - \partial^{\beta\gamma}\rho_{(1)}^{\alpha\gamma}(x)c_+^0(\beta_L^0)^2) |0\rangle_{R-R}^{\alpha\beta} \quad (2.6)$$

where $\rho_{(\pm)}^{\alpha\beta} = \rho_{(0)}^{\alpha\beta} \pm \rho_{(1)}^{\alpha\beta}$. The constant modes of $A_{(\pm)}^{\alpha\beta}$ are in the BRST cohomology since $|0\rangle_{R-R}^{\alpha\beta}$ and $\beta_L^0\gamma_R^0|0\rangle_{R-R}^{\alpha\beta}$ can not be written as $Q|\Lambda\rangle$ if $|\Lambda\rangle$ is restricted to contain a finite number of terms. (If an infinite number of terms are allowed, $|0\rangle_{R-R}^{\alpha\beta} = Q\sum_{n=0}^{\infty} (-1)^n c_+^0 (\beta_L^0)^{2+2n} (\gamma_R^0)^{2n} |0\rangle_{R-R}^{\alpha\beta}$ and $\beta_L^0\gamma_R^0|0\rangle_{R-R}^{\alpha\beta} = Q\sum_{n=0}^{\infty} (-1)^n c_+^0 (\beta_L^0)^{3+2n} (\gamma_R^0)^{1+2n} |0\rangle_{R-R}^{\alpha\beta}$.) One way to see the necessity of this restriction on $|\Lambda\rangle$ is to note that $\langle D|0\rangle_{R-R}^{\alpha\beta}$ is non-zero where $\langle D|$ is the D -brane boundary state[9]. Since $\langle D|Q = 0$, $|0\rangle_{R-R}^{\alpha\beta}$ can not be BRST-trivial.[10]

Note that these constant modes of $A_{(\pm)}^{\alpha\beta}$ would not be present if one had instead chosen the R-R vacuum to be annihilated by β_L^0 and β_R^0 . In such a picture, the only massless R-R state would be $F^{\alpha\beta}$ with the equations of motion $\partial^{\alpha\gamma}F^{\gamma\beta} = \partial^{\beta\gamma}F^{\alpha\gamma} = 0$. However, in this case, there is no action in terms of just $F^{\alpha\beta}$ (i.e. without gauge fields) which can reproduce these equations of motion. This is especially problematic for the Type IIB superstring where one needs an infinite number of auxiliary fields to write an action for the self-dual five-form field strength.[4][11]

2.2. The massless NS-NS, NS-R and R-NS sectors

The NS-NS, NS-R, and R-NS sectors of the Type II superstring are similarly described by a string field constructed from non-negative modes acting on a vacuum. The vacuum $|0\rangle_{NS-NS}$ will be defined in the picture $(P_L, P_R) = (-1, -1)$, $|0\rangle_{NS-R}^{\alpha}$ will be defined in the picture $(P_L, P_R) = (-1, -\frac{1}{2})$, and $|0\rangle_{R-NS}^{\alpha}$ will be defined in the picture $(P_L, P_R) = (-\frac{3}{2}, -1)$. So like $|0\rangle_{R-R}^{\alpha\beta}$, these vacua are annihilated by all negative modes and by β_R^0 , γ_L^0 and $b_{L/R}^0$. In terms of the SL(2) invariant vacuum, these vacua are $c_L e^{-\phi_L} c_R e^{-\phi_R}$, $c_L e^{-\phi_L} c_R e^{-\frac{1}{2}\phi_R}$, and $c_L e^{-\frac{3}{2}\phi_L} c_R e^{-\phi_R}$.

The massless states in these sectors are described by

$$|\Phi\rangle_{NS-NS} = ((g_{\mu\nu}(x) + b_{\mu\nu}(x))\psi_L^{\frac{1}{2}\mu}\psi_R^{\frac{1}{2}\nu} + \phi(x)\beta_L^{\frac{1}{2}}\gamma_R^{\frac{1}{2}} + \tilde{\phi}(x)\gamma_L^{\frac{1}{2}}\beta_R^{\frac{1}{2}}) \quad (2.7)$$

$$\begin{aligned}
& +B_\mu(x)c_+^0\beta_L^{\frac{1}{2}}\psi_R^{\frac{1}{2}\mu} + \tilde{B}_\mu(x)c_+^0\psi_L^{\frac{1}{2}\mu}\beta_R^{\frac{1}{2}} |0\rangle_{NS-NS}, \\
|\Phi\rangle_{NS-R} &= (\chi_\mu^\alpha(x)\psi_L^{\frac{1}{2}\mu} + \tau^\alpha(x)c_+^0\beta_L^{\frac{1}{2}} + \Xi^\alpha(x)\beta_L^{\frac{1}{2}}\gamma_R^0) |0\rangle_{NS-R}^\alpha, \\
|\Phi\rangle_{R-NS} &= (\tilde{\chi}_\mu^\alpha(x)c_+^0\beta_L^0\psi_R^{\frac{1}{2}\mu} + \tilde{\tau}^\alpha(x)c_+^0\beta_R^{\frac{1}{2}} + \tilde{\Xi}^\alpha(x)c_+^0(\beta_L^0)^2\gamma_R^{\frac{1}{2}} \\
& + M_\mu^\alpha(x)\psi_R^{\frac{1}{2}\mu} + N^\alpha(x)\beta_L^0\gamma_R^{\frac{1}{2}}) |0\rangle_{R-NS}^\alpha.
\end{aligned}$$

Note that $[B^\mu, \tilde{B}^\mu, \tau^\alpha, \tilde{\tau}^\alpha]$ are auxiliary fields, and $[\tilde{\phi}, M_\mu^\alpha, N^\alpha]$ can be eliminated by algebraic gauge transformations.

The remaining fields transform under coordinate reparameterizations, $b_{\mu\nu}$ gauge transformations, and N=2 local supersymmetry transformations as

$$\delta(g_{\mu\nu} + b_{\mu\nu}) = \partial_\mu y_\nu + \partial_\nu \tilde{y}_\mu, \quad \delta\phi = \partial^\mu(y_\mu + \tilde{y}_\mu), \quad (2.8)$$

$$\delta\chi_\mu^\alpha = \partial_\mu\epsilon^\alpha, \quad \delta\tilde{\chi}_\mu^\alpha = \partial_\mu\tilde{\epsilon}^\alpha, \quad \delta\Xi^\alpha = \partial^{\alpha\beta}\epsilon^\beta, \quad \delta\tilde{\Xi}^\alpha = \partial^{\alpha\beta}\tilde{\epsilon}^\beta,$$

which are parameterized by the gauge field

$$|\Lambda\rangle_{NS-NS} = (y_\mu(x)\beta_L^{\frac{1}{2}}\psi_R^{\frac{1}{2}\mu} + \tilde{y}_\mu(x)\psi_L^{\frac{1}{2}\mu}\beta_R^{\frac{1}{2}} - \partial^\mu\tilde{y}_\mu c_+^0\beta_L^{\frac{1}{2}}\beta_R^{\frac{1}{2}}) |0\rangle_{NS-NS}, \quad (2.9)$$

$$|\Lambda\rangle_{NS-R} = \epsilon^\alpha(x)\beta_L^{\frac{1}{2}} |0\rangle_{NS-R}^\alpha,$$

$$|\Lambda\rangle_{R-NS} = \tilde{\epsilon}^\alpha(x)(c_+^0\beta_L^0\beta_R^{\frac{1}{2}} - c_+^0(\beta_L^0)^3\gamma_R^{\frac{1}{2}}) |0\rangle_{R-NS}^\alpha.$$

Note that these transformations have been chosen to preserve the gauge $\tilde{\phi} = M_\mu^\alpha = N^\alpha = 0$.

3. Conserved Charges

When the gauge parameters $[y_\mu, \tilde{y}_\mu, \epsilon^\alpha, \tilde{\epsilon}^\alpha, \rho_{(0)}^{\alpha\beta}, \rho_{(1)}^{\alpha\beta}]$ of equations (2.9) and (2.3) are constants, $Q|\Lambda\rangle = 0$. The -1 ghost-number states described by $|\Lambda\rangle$ are

$$\begin{aligned}
P_R^\mu &= \beta_L^{\frac{1}{2}}\psi_R^{\frac{1}{2}\mu} |0\rangle_{NS-NS}, \quad P_L^\mu = \psi_L^{\frac{1}{2}\mu}\beta_R^{\frac{1}{2}} |0\rangle_{NS-NS} \\
q_R^\alpha &= \beta_L^{\frac{1}{2}}|0\rangle_{NS-R}^\alpha, \quad q_L^\alpha = (c_+^0\beta_L^0\beta_R^{\frac{1}{2}} - c_+^0(\beta_L^0)^3\gamma_R^{\frac{1}{2}}) |0\rangle_{R-NS}^\alpha, \\
C_{(0)}^{\alpha\beta} &= \beta_L^0 |0\rangle_{R-R}^{\alpha\beta}, \quad C_{(1)}^{\alpha\beta} = (\beta_L^0)^2\gamma_R^0 |0\rangle_{R-R}^{\alpha\beta},
\end{aligned} \quad (3.1)$$

which are the conserved charges associated with the global part of the gauge transformations. These states cannot be written as $Q|\Omega\rangle$ where $|\Omega\rangle$ has a finite number of terms, so they are in the BRST cohomology at -1 ghost number.

Note that the conserved R-R charges, $C_{(0)}^{\alpha\beta}$ and $C_{(1)}^{\alpha\beta}$, are in the same cohomology class as the states $(\beta_L^0)^n(\gamma_R^0)^{n-1}|0\rangle_{R-R}^{\alpha\beta}$ where n is an arbitrarily large odd or even number. This means that the R-R charges act as BRST-trivial operators on any superstring state which carries finite left-moving ghost number. This includes all perturbative superstring states, but not D -brane boundary states which contain the ghost dependence $e^{\beta_L^0\gamma_R^0}$. [12] For this reason, D -branes can carry non-zero R-R charge but perturbative superstring states cannot. Since R-R charge is related to momentum in the eleventh direction, there is a relation between non-trivial dependence on worldsheet ghosts and non-trivial dependence on the eleventh direction.

By hitting with the left and right-moving picture-changing operators, $Z_L Z_R$, it is easy to see that the $NS - NS$ charges are the $(P_L, P_R) = (-1, -1)$ versions of the translation and NS-NS one-brane generators $\int d\sigma(\partial_\tau x^\mu \pm \partial_\sigma x^\mu)$, and the $NS - R$ and $R - NS$ charges are the $(-3/2, -1)$ and $(-1, -1/2)$ versions of the $N=2$ supersymmetry generators $q_{L/R}^\alpha = \int d\sigma e^{-\frac{1}{2}\phi_{L/R}} \Sigma_{L/R}^\alpha$. However, $Z_L Z_R$ annihilates the R-R charges (see footnote 3) so one needs to develop a “picture-raising” prescription which preserves BRST cohomology. This has been accomplished in [13] where it will be shown that the R-R charge $C_{(0)}^{\alpha\beta}$ of (3.1) is the $(P_L, P_R) = (-3/2, -1/2)$ version of the R-R charge $\int d\sigma \lambda_L^\alpha \lambda_R^\beta$ which was proposed in reference [14] using a twistor-like construction.

4. Supersymmetry Algebra of the Superstring

Finally, it will be shown that the supersymmetry transformations of the superstring close to an $N=2$ SUSY algebra including R-R central charges. Since the SUSY generators carry picture, one needs to be careful to choose the right picture when defining the supersymmetry transformations of the string field. For a global supersymmetry transformation parameterized by u_α and \tilde{u}_α , the correct choice is

$$\delta|\Phi\rangle_{NS-NS} = \tilde{u}^\alpha \bar{q}_L^\alpha |\Phi\rangle_{R-NS} + u^\alpha q_R^\alpha |\Phi\rangle_{NS-R}, \quad (4.1)$$

$$\delta|\Phi\rangle_{R-NS} = \tilde{u}^\alpha q_L^\alpha |\Phi\rangle_{NS-NS} + u^\alpha q_R^\alpha |\Phi\rangle_{R-R},$$

$$\delta|\Phi\rangle_{NS-R} = \tilde{u}^\alpha \bar{q}_L^\alpha |\Phi\rangle_{R-R} + u^\alpha \bar{q}_R^\alpha |\Phi\rangle_{NS-NS},$$

$$\delta|\Phi\rangle_{R-R} = \tilde{u}^\alpha q_L^\alpha |\Phi\rangle_{NS-R} + u^\alpha \bar{q}_R^\alpha |\Phi\rangle_{R-NS},$$

where $q_{L/R}^\alpha$ carries $-1/2$ picture and $\bar{q}_{L/R}^\alpha$ carries $+1/2$ picture. The easiest way to define the action of $q_{L/R}^\alpha$ or $\bar{q}_{L/R}^\alpha$ on the string field is to first write the string field in terms

of the SL(2)-invariant vacuum, then take the contour integral of $\int d\sigma e^{-\frac{1}{2}\phi_{L/R}\Sigma_{L/R}^\alpha}$ or $\int d\sigma(e^{\frac{1}{2}\phi_{L/R}\Gamma_\mu^{\alpha\beta}\Sigma_{L/R}^\beta}\partial_{L/R}x^\mu + b\eta e^{\frac{3}{2}\phi_{L/R}\Sigma_{L/R}^\alpha})$ around the vertex operator, and finally re-express the resulting vertex operator in terms of the string vacuum. Note that in the gauge where $\tilde{\phi} = M_\mu^\alpha = N^\alpha = A_{(2)}^{\alpha\beta} = 0$, one also needs to include compensating gauge transformations which cancel the supersymmetry transformation of $\tilde{\phi}$, M_μ^α , N^α and $A_{(2)}^{\alpha\beta}$. For example, $\delta|\Phi\rangle_{NS-NS}$ needs to include the term $Q\tilde{u}^\alpha\tilde{\tau}^\alpha(x) c_+^0\beta_L^{\frac{1}{2}}\beta_R^{\frac{1}{2}}|0\rangle_{NS-NS}$ to cancel the variation of $\tilde{\phi}$, and $\delta|\Phi\rangle_{R-R}$ needs to include the term $Qu^\beta\tilde{\Xi}^\alpha(x) c_+^0(\beta_L^0)^2|0\rangle_{R-R}^{\alpha\beta}$ to cancel the variation of $A_{(2)}^{\alpha\beta}$.

The simplest non-trivial computation is the field-independent part of the transformation resulting from the commutator of a global supersymmetry transformation with a local supersymmetry transformation. (The field-independent part vanishes for the commutator of two global SUSY transformations, and is complicated for the commutator of two local SUSY transformations.)

Using (4.1) and (2.9), the field-independent part of this commutator is

$$\begin{aligned}\delta|\Phi\rangle_{NS-NS} &= \tilde{u}^\alpha\bar{q}_L^\alpha Q\tilde{\epsilon}^\beta c_+^0(\beta_L^0\beta_R^{\frac{1}{2}} - (\beta_L^0)^3\gamma_R^{\frac{1}{2}})|0\rangle_{R-NS}^\beta \\ &+ u^\alpha q_R^\alpha Q\epsilon^\beta\beta_L^{\frac{1}{2}}|0\rangle_{NS-R}^\beta + Q\tilde{u}^\alpha\partial^{\alpha\gamma}\tilde{\epsilon}^\gamma c_+^0\beta_L^{\frac{1}{2}}\beta_R^{\frac{1}{2}}|0\rangle_{NS-NS}, \\ \delta|\Phi\rangle_{NS-R} &= \delta|\Phi\rangle_{R-NS} = 0, \\ \delta|\Phi\rangle_{R-R} &= \tilde{u}^\alpha q_L^\alpha Q\epsilon^\beta\beta_L^{\frac{1}{2}}|0\rangle_{NS-R}^\beta + u^\alpha\bar{q}_L^\alpha Q\tilde{\epsilon}^\beta c_+^0(\beta_L^0\beta_R^{\frac{1}{2}} - (\beta_L^0)^3\gamma_R^{\frac{1}{2}})|0\rangle_{R-NS}^\beta \\ &+ Qu^\beta\partial^{\alpha\gamma}\tilde{\epsilon}^\gamma c_+^0(\beta_L^0)^2|0\rangle_{R-R}^{\alpha\beta},\end{aligned}\tag{4.2}$$

where $[u^\alpha, \tilde{u}^\alpha]$ are global parameters and $[\epsilon^\alpha(x), \tilde{\epsilon}^\alpha(x)]$ are local parameters.

Converting to the SL(2)-invariant vacuum to compute the action of $q_{L/R}^\alpha$ and $\bar{q}_{L/R}^\alpha$, and then re-expressing in terms of the original vacuum, one finds

$$\begin{aligned}\delta|\Phi\rangle_{NS-NS} &= \Gamma_{\alpha\beta}^\mu Q(u^\alpha\epsilon^\beta\beta_L^{\frac{1}{2}}\psi_R^{\frac{1}{2}\mu} + \tilde{u}^\alpha\tilde{\epsilon}^\beta\psi_L^{\frac{1}{2}\mu}\beta_R^{\frac{1}{2}} + \tilde{u}^\alpha\partial^{\beta\gamma}\tilde{\epsilon}^\gamma c_+^0\beta_L^{\frac{1}{2}}\beta_R^{\frac{1}{2}})|0\rangle_{NS-NS}, \\ \delta|\Phi\rangle_{NS-R} &= \delta|\Phi\rangle_{R-NS} = 0, \\ \delta|\Phi\rangle_{R-R} &= Q(\tilde{u}^\alpha\epsilon^\beta + u^\beta\tilde{\epsilon}^\alpha)\beta_L^0|0\rangle_{R-R}^{\alpha\beta}.\end{aligned}\tag{4.3}$$

So the field-independent part of the transformation resulting from the commutator of a local and global supersymmetry transformation is given by

$$[\delta_{q_L^\alpha}(\tilde{\epsilon}^\alpha(x)) + \delta_{q_R^\alpha}(\epsilon^\alpha(x)), \delta_{q_L^\beta}(\tilde{u}^\beta) + \delta_{q_R^\beta}(u^\beta)]\tag{4.4}$$

$$= \delta_{P_R^\mu}(\epsilon^\alpha(x)\Gamma_{\alpha\beta}^\mu u^\beta) + \delta_{P_L^\mu}(\tilde{\epsilon}^\alpha(x)\Gamma_{\alpha\beta}^\mu \tilde{u}^\beta) + \delta_{C_{(0)}^{\alpha\beta}}(\tilde{u}^\alpha \epsilon^\beta(x) + u^\beta \tilde{\epsilon}^\alpha(x))$$

where $\delta_{P_R^\mu}(y^\mu)$ and $\delta_{P_L^\mu}(\tilde{y}^\mu)$ are NS-NS gauge transformations parameterized by y^μ and \tilde{y}^μ of equation (2.9), $\delta_{q_R^\alpha}(\epsilon^\alpha)$ and $\delta_{q_L^\alpha}(\tilde{\epsilon}^\alpha)$ are R-NS and NS-R gauge transformations parameterized by ϵ^α and $\tilde{\epsilon}^\alpha$ of equation (2.9), and $\delta_{C_{(0)}^{\alpha\beta}}(\rho_{(0)}^{\alpha\beta})$ are R-R gauge transformations parameterized by $\rho_{(0)}^{\alpha\beta}$ of equation (2.3). Therefore, supersymmetry transformations of the Type II superstring form an N=2 SUSY algebra with an NS-NS one-brane central charge and with 256 R-R central charges.

Note that, unlike the conjecture of [14], there is no e^ϕ dependence in $\{q_L^\alpha, q_R^\beta\}$. Nevertheless, the claim is still correct in [14] that momentum in the eleventh direction, P_{11} , should be associated with e^ϕ times the R-R central charge $C_{(0)}^{\alpha\alpha}$. The e^ϕ dependence comes from the fact that the N=1 D=11 SUSY algebra is

$$\{\hat{q}^A, \hat{q}^B\} = \Gamma_M^{AB} E^{Mm} P_m \quad (4.5)$$

where $A = 1$ to 32 are SO(10,1) spinor indices, $M = 0, \dots, 9, 11$ are flat vector indices, $m = 0, \dots, 9, 11$ are curved vector indices, \hat{q}^A are the N=1 D=11 SUSY generators, and E^{Mm} is the D=11 vierbein. When the D=10 metric is flat, $E^{Mm} = e^{\frac{1}{3}\phi} \delta^{Mm}$ for $M=0$ to 9 and $E^{11\ m} = e^{-\frac{2}{3}\phi} \delta^{11\ m}$. [15] So if P_m is the ten-dimensional momentum for $m = 0$ to 9, then $\hat{q}^A = e^{\frac{1}{6}\phi} q_L^\alpha$ for $A = 1$ to 16 and $\hat{q}^A = e^{\frac{1}{6}\phi} q_R^\alpha$ for $A = 17$ to 32. Comparing with (4.4), this implies that $P_{11} = \frac{1}{32} e^\phi C_{(0)}^{\alpha\alpha}$.

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